Optomechanical photon detection and enhanced dispersive phonon readout

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In cavity optomechanics, nanomechanical motion couples to a localized optical mode. The regime of single-photon strong coupling is reached when the optical shift induced by a single phonon becomes comparable to the cavity linewidth. We consider a setup in this regime comprising two optical modes and one mechanical mode. For mechanical frequencies nearly resonant to the optical level splitting, we find the photon-phonon and the photon-photon interactions to be significantly enhanced. In addition to dispersive phonon detection in a novel regime, this offers the prospect of optomechanical photon measurement. We study these QND detection processes using both analytical and numerical approaches.

Introduction. - By coupling mechanical resonators to the light of optical cavities the emerging field of optomechanics [1] aims at observing quantum mechanical behavior of macroscopic systems. The ultimate goal is the regime where single phonons and photons interact strongly. New architectures and progress in design and fabrication pave the way towards realizing strong coupling even at the single-photon level in optomechanical systems [2–7]. This development has stimulated several theoretical works that analyze the generic optomechanical system, i.e. a single optical mode coupled to a single mechanical mode, in the regime of strong coupling. Nonclassical effects are found in the dynamics of the mechanical resonator [8–10] and the statistics of the light field [9, 11] if the photon-phonon coupling rate g_0 becomes comparable to both the decay rate of the cavity κ and the mechanical oscillation frequency Ω.

A fundamental test of the quantum nature of a mechanical resonator would be an observation of energy quantization and quantum jumps between Fock states. In a pioneering work, Thompson *et al.* [12] presented the possibility of a quantum non-demolition (QND) measurement of the phonon number in a modified optomechanical setup comprising two cavity modes separated by a dielectric mechanical membrane, see Fig.1(a). Subsequently the requirements for the experimental parameters and the limitations imposed by quantum noise have been studied in [13–15]. An enhancement of the nonlinear coupling by making use of the full spectrum of transverse cavity modes has been demonstrated in [16]. However, the analysis of the double cavity optomechanical setup has so far been restricted to cases, where the influence of individual photons is weak. Furthermore, it was assumed that the mechanical and optical timescales separate, i.e. that the mechanical frequency is much smaller than the optical level splitting. Hence this description is not suited for the ultimate quantum regime.

In this article we present a general framework for the double cavity setup in terms of an effective Hamiltonian that captures the regime of strong optomechanical coupling $g_0/\kappa \gtrsim 1$ and large mechanical frequencies. In our analysis the difference between optical level splitting and mechanical frequency, $\delta\Omega = 2J - \Omega$, appears as a crucial parameter. It enters the coupling rate $g_0^2/\delta\Omega$ that characterizes the coherent interaction among photons and between photons and phonons. If this dispersive optical frequency shift exceeds the cavity decay rate, one enters what we will call the strong dispersive coupling regime: $g_0^2 \gtrsim \kappa\delta\Omega$. Since $\delta\Omega$ can be made much smaller than Ω, this condition is easier to achieve than the corresponding one for the generic optomechanical system, $g_0^2 \gtrsim \kappa\Omega$. This is relevant in particular because optomechanical systems have by now reached the regime of large mechani-

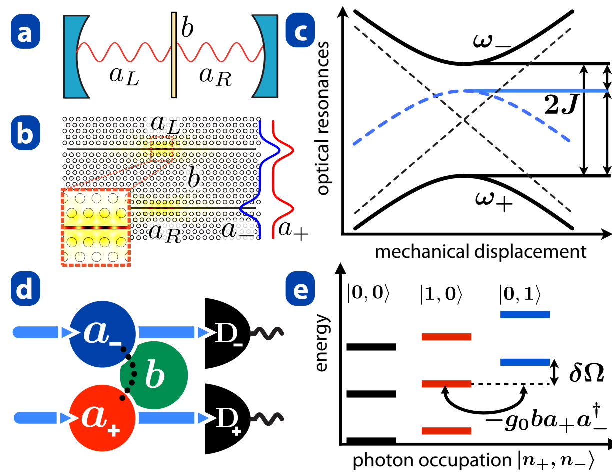

Figure 1. (a),(b) Implementations of the double cavity setup: membrane in the middle (a) and optomechanical crystal setup (b). (c) Optical resonances as a function of mechanical displacement. (d) Scheme depicting the mechanical mode (b) and the optical modes $a_\pm$. The cavities are driven by independent laser sources and the transmitted signal is measured by photodetectors $D_\pm$. (e) Energy level scheme of the double cavity optomechanical system: For $\delta\Omega = 2J - \Omega \ll \Omega$ the transitions induced by $ba_+a_-^\dagger$ are energetically favorable (as compared to $b^\dagger a_+^\dagger a_-$).

cal frequencies, see for example [5–7], where they are less susceptible to thermal fluctuations and optomechanical cooling is more efficient.

For mechanical frequencies comparable to the optical splitting, i.e. $\delta\Omega \ll J, \Omega$, the phonon-photon interaction renders possible a QND phonon measurement with an enhanced optical frequency shift of $g_0^2/\delta\Omega$, as compared to the result for low mechanical frequencies of g_0^2/J. As a completely new feature, the photon-photon interaction makes available a QND measurement of the photon number of one of the optical modes by detecting the other one. We analyze the prospects of both phonon and photon Fock state detection and their limitations due to quantum noise and present data from numerical simulations of the dissipative quantum dynamics.

Model. - We consider an optomechanical setup consisting of two optical modes ($a_\pm$, frequencies $\omega_\pm$) and one mechanical mode (b, frequency Ω) that is described by a Hamiltonian

$$H = H_0 + H_{\text{int}} + H_{\text{drive}} + H_{\text{diss}}, \tag{1}$$

$$H_0 = \omega_- a_-^\dagger a_- + \omega_+ a_+^\dagger a_+ + \Omega b^\dagger b \tag{2}$$

$$H_{\text{int}} = -g_0 (b^\dagger + b)(a_+^\dagger a_- + a_-^\dagger a_+) \tag{3}$$

$$H_{\text{drive}} = \alpha_\pm (e^{i\omega_{L\pm} t} a_\pm + H.c.) \tag{4}$$

The optomechanical coupling rate is denoted by g_0, and both optical modes are pumped by laser sources at rates $\alpha_\pm$. The optical cavities are characterized by the photon decay rates into the reflection channel ($\kappa_{\pm,r}$) and into the transmission channel ($\kappa_{\pm,t}$) with $\kappa_\pm = \kappa_{\pm,r} + \kappa_{\pm,t}$. We assume that the transmitted signal from each of the modes can be filtered and measured independently using a photodetector ($D_\pm$), see Fig. 1(d). The mechanical resonator couples to a thermal bath at a rate Γ with a bath occupation given by n_{th}. In the following, we assume the mechanical frequency to be high enough and the bath temperature to be low enough such that the oscillator is sufficiently close to the ground state.

A Hamiltonian of the form of Eq. (1) is found both in the "membrane in the middle"-setup [12] and in optomechanical crystals [17]. The optical modes $a_\pm$ constitute normal modes $a_\pm = (a_L \pm a_R)/\sqrt{2}$, where $a_{L,R}$ denotes geometrically distinct optical modes with an original Hamiltonian $\tilde{H} = \tilde{H}_0 + \tilde{H}_{\text{int}}$, where

$$\tilde{H}_{\text{int}} = -J(a_L^\dagger a_R + H.c.) - g_0(b^\dagger + b)(a_L^\dagger a_L - a_R^\dagger a_R) \tag{5}$$

and $\tilde{H}_0 = \omega(a_L^\dagger a_L + a_R^\dagger a_R) + \Omega b^\dagger b$. The frequency splitting of the normal modes is thus given by the photon tunnel coupling rate J, $\omega_- - \omega_+ = 2J$.

In the approach of [12–15] the optical resonances are calculated as $\omega \pm \sqrt{J^2 + (g_0 \tilde{x})^2} \approx \omega_\pm \pm \frac{g_0^2}{2J}\tilde{x}^2$ (see Fig. 1(c)), where $\tilde{x} = b^\dagger + b$ is the mechanical displacement in units of the mechanical ground state width and where it is assumed that $J \gg g_0 \tilde{x}$. Note that $\tilde{x}$ is treated as a quasistatic variable (in the sense of the Born-Oppenheimer approximation, with photons playing the role of electrons). This approach therefore has to fail if the optical frequency splitting and the mechanical excitation energy become comparable.

Effective Description. - A general description in terms of an effective Hamiltonian $H_{\text{eff}} = e^{iS}(H_0 + H_{\text{int}})e^{-iS}$ is obtained by applying a unitary transformation with $S = \frac{ig_0}{2J-\Omega}(ba_+a_-^\dagger - H.c.) + \frac{ig_0}{2J+\Omega}(b^\dagger a_+ a_-^\dagger - H.c.)$. To second order in g_0 the effective Hamiltonian reads

$$H_{\text{eff}} = H_0 + \frac{g_0^2}{2}\Big(\frac{1}{2J-\Omega} + \frac{1}{2J+\Omega}\Big)\Big(n_- - n_+\Big)\Big(b^\dagger + b\Big)^2 + \frac{g_0^2}{2}\Big(\frac{1}{2J-\Omega} - \frac{1}{2J+\Omega}\Big)\Big(a_+^\dagger a_- + a_+ a_-^\dagger\Big)^2 \tag{6}$$

where $n_\pm = a_\pm^\dagger a_\pm$ and where we disregard terms of order $g_0^3/\delta\Omega^2$. For the most interesting regime of mechanical frequencies comparable to the optical splitting, i.e. $\delta\Omega = 2J - \Omega \ll J, \Omega$ the leading contribution is given by

$$H_{\text{eff}} = H_0 + \frac{g_0^2}{\delta\Omega}\big(n_+ n_- + n_- + n_- n_b - n_+ n_b\big) \tag{7}$$

where $n_b = b^\dagger b$ and where we neglect terms of the order $g_0^2/(2J+\Omega)$ and rapidly rotating terms like $b^{\dagger 2}$, $(a_+^\dagger a_-)^2$.

Phonon detection. - The effective Hamiltonian of Eq. (6) enables us to discuss optomechanical QND phonon detection in its most general form, going beyond previous discussions [13–15]. The optical frequencies are shifted by $\mp g_0^2(\frac{1}{2J-\Omega} + \frac{1}{2J+\Omega})n_b$, proportional to the phonon number n_b. We note that in the limit $\Omega \ll J$ the result of [13] is recovered. However, for mechanical frequencies comparable to the optical splitting, i.e. $\delta\Omega = 2J - \Omega \ll 2J$, the frequency shift per phonon $\delta\omega = g_0^2/\delta\Omega$ is strongly enhanced. We stress that the enhancement of the frequency shift is observable even in the weak coupling regime $g_0 \ll \kappa_\pm$, where the cavity modes have to be strongly driven in order to detect the frequency shift in a homodyne measurement scheme [12, 15]. In the following, however, we focus on the regime where both $\Omega \approx 2J$ and $g_0 \gtrsim \kappa_\pm$ and where single quanta affect the optical and mechanical modes strongly.

The experimental protocol for detecting the phonon number is to pump one of the optical modes (here a_+) with a laser at frequency ω_{L+} and measure the transmitted signal using a photodetector (D_+). The second mode (a_-) is undriven, playing the role of an idle spectator (though it will become important for dissipative processes, see below). We first study the spectrum of the detection mode a_+, i.e. the photon number $\bar{n}_+$ as a function of detuning $\omega_{L+} - \omega_+$. In steady state, the spectrum consists of several resonances with spacing $\delta\omega$ corresponding to different phonon number states. In a situation where the optical frequency shift per phonon $\delta\omega$ is smaller than the cavity linewidth κ_+, the resonances overlap, see Fig. 2(a). In the following section, we will discuss this "weak dispersive coupling" regime (even though g_0/κ will still be taken on the order of one). Note that the strong dispersive regime is also relevant, both for phonon and photon detection, and we will come back to it when discussing photon measurements. The time evolution of the mechanical state can be monitored by pumping the detection mode at fixed detuning and recording the photon counts at the detector during an interval τ_{meas}. A quantum jump in the phonon number results in a change of the intracavity photon number by $\Delta\bar{n}_+ \approx \bar{n}_+\delta\omega/\kappa_+$ and a change of photon counts at D_+ by $\kappa_{+,t}\Delta\bar{n}_+\tau_{\text{meas}}$. The measurement time τ_{meas} has to be chosen large enough, such that the measured signal exceeds the photon number uncertainty, i.e. $\Delta\bar{n}_+\kappa_{+,t}\tau_{\text{meas}} > \sqrt{\bar{n}_+\kappa_{+,t}\tau_{\text{meas}}}$ [18] or equivalently:

$$\tau_{\text{meas}} > \frac{\kappa_+^2/\kappa_{+,t}}{\delta\omega^2\bar{n}_+}. \tag{8}$$

On the other hand, the measurement time has to be smaller than the lifetime of a phonon Fock state which is governed by thermal fluctuations at rate Γ_{th} and by decoherence induced via the optical modes at rate Γ_{ind}:

$$\max\{\Gamma_{\text{th}}, \Gamma_{\text{ind}}\}\,\tau_{\text{meas}} < 1. \tag{9}$$

The thermalization rate of the phonon state $\bar{n}_b$ is given by $\Gamma_{\text{th}} = \Gamma\big((n_{\text{th}}+1)\bar{n}_b + n_{\text{th}}(\bar{n}_b+1)\big)$ in the uncoupled system. The major contribution to Γ_{ind} stems from the process where a phonon is annihilated while a photon tunnels from the a_+ to the a_- mode and decays. A calculation according to Fermi's golden rule yields $\Gamma_{\text{ind}} \approx g_0^2\bar{n}_+\bar{n}_b\kappa_-/\delta\Omega^2$. It follows that single-photon strong coupling, i.e. $g_0^2 > \kappa_+\kappa_-$, is required to obtain a signal to noise ratio bigger than one, as has already been shown by [14] for the limiting case of small mechanical frequencies $\Omega \ll J$. We note that a phonon measurement using the a_- mode for detection can be described analogously, the main qualitative difference being that the cavity-induced decoherence processes excite phonons and potentially cause an instability.

To simulate the envisaged QND phonon measurement, we employ the Lindblad master equation for the system's density matrix ρ,

$$\frac{d}{dt}\rho = -i[H,\rho] + \sum_{\text{unobserved}} \mathcal{D}[c_i]\rho + \sum_{\text{observed}} \mathcal{D}[d_i]\rho \tag{10}$$

where $\mathcal{D}[A]\rho = A\rho A^\dagger - \frac{1}{2}A^\dagger A\rho - \frac{1}{2}\rho A^\dagger A$. The unobserved channels are the photon decay into the reflection channels $c_{3,4} = \sqrt{\kappa_{\pm,r}}a_\pm$ and the coupling between the mechanical resonator and the thermal environment with $c_1 = \sqrt{\Gamma(n_{\text{th}}+1)}b$ and $c_2 = \sqrt{\Gamma n_{\text{th}}}b^\dagger$, while the transmission channels $d_{1,2} = \sqrt{\kappa_{\pm,t}}a_\pm$ are under observation.

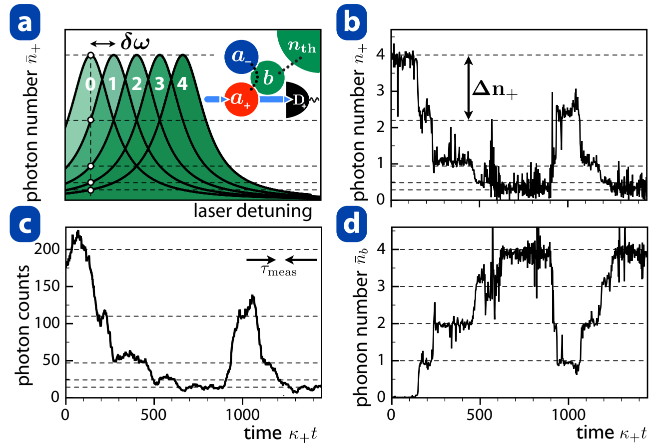

Figure 2. Phonon detection in the weak dispersive coupling regime $\delta\omega = g_0^2/\delta\Omega < \kappa_+$, for single-photon strong coupling $g_0/\kappa_+ = 3$: (a) Schematic illustration of the resonances of the detection mode corresponding to phonon number states 0,1,2,3,4. A jump between phonon Fock states can be detected if a difference Δn_+ in the intracavity photon number is resolved. (b),(d) Quantum trajectories of the photon number in the detection mode, $\bar{n}_+ = \langle a_+^\dagger a_+\rangle$ (b), and the phonon number, $\bar{n}_b = \langle b^\dagger b\rangle$ (d) from a numerical simulation of the stochastic master equation. $g_0 = 3\kappa_+$, $\delta\Omega = 20\kappa_+$, $n_{\text{th}} = 2$, $\Gamma = 10^{-3}\kappa_+$, $\alpha_+ = \kappa_+$, $\Delta_+ = 0$, $\kappa_- = 10^{-2}\kappa_+$. (c) Photon counts recorded at the photodetector D_+ within an interval $[t-\tau_{\text{meas}}, t]$, where a measurement time of $\tau_{\text{meas}} = 50\kappa_+^{-1}$ considerably smaller than the lifetime of a phonon state was chosen ($\Gamma_{\text{th}}^{-1} \approx 140\kappa_+^{-1}$, $\Gamma_{\text{ind}}^{-1} \approx 1100\kappa_+^{-1}$).

We unravel the time evolution into quantum jumps [19] $\rho(t+dt) = d_i\rho(t)d_i^\dagger/\langle d_i^\dagger d_i\rangle(t)$ that occur with probability $p_i(t) = \gamma_i\langle d_i^\dagger d_i\rangle(t)dt$ and into the deterministic part $\rho(t+dt) = \rho(t) - (i[H,\rho(t)] - \sum_i \mathcal{D}[c_i]\rho(t))dt + \sum_i\{\gamma d_i^\dagger d_i/2, \rho(t)\}dt$ plus subsequent normalization. A quantum jump with $d_{1,2} = \sqrt{\kappa_{\pm,t}}a_\pm$ is interpreted as a detection event at the photodetector D_+ or D_-, respectively. Figure 2 (b)-(d) shows trajectories from such a simulation. The phonon number jumps between the Fock states 0 and 4, driven by thermal fluctuations (Fig. 2d). The photon number in the detection mode follows the time evolution of the mechanical mode (Fig. 2b). Thus, by monitoring the photon counts at the photodetector (Fig. 2c) a QND measurement of the phonon number is achieved, with a signal strongly enhanced with respect to the previously considered configuration $\Omega \ll J$.

Photon detection. - As a novel feature of the system, we identify the dispersive photon-photon interaction in the effective Hamiltonian (7). We note that the interaction term vanishes in the limit of small mechanical frequencies $\Omega \ll J$ and therefore did not appear in previous works. Here we demonstrate the prospects of a QND measurement of the photon number n_+ using the a_- mode for detection. The roles of the two optical modes are chosen as to suppress the influence of unwanted transitions from the a_- mode to the energetically lower-lying a_+ mode. Both modes are driven independently by a laser and the data from the photodetector D_- is used to extract the information about the photon number n_+. We assume that the detection mode has a lower finesse than the signal mode, i.e. $\kappa_- \gg \kappa_+$, such that a sufficiently large number of photons arrives at D_- while the state of a_+ is only weakly perturbed by the photons in a_-.

In the weak dispersive coupling regime, $g_0^2 < \kappa_- \delta\Omega$, we find a required measurement time of

$$\tau_{\text{meas}} > \frac{\kappa_-^2/\kappa_{-,t}}{\delta\omega^2 \bar{n}_-}$$

with a frequency shift per photon of $\delta\omega = g_0^2/\delta\Omega$, in analogy to the case of phonon detection discussed above (see also Fig. 2). In order to detect the photon state $\bar{n}_+$ within its lifetime, it is also required that $\tau_{\text{meas}} < 1/\bar{n}_+\kappa_+$. Moreover, the measurement would be spoiled if a phonon were to be excited during the measurement time, since a_- actually measures $n_+ + n_b$. We therefore demand that both the thermalization rate Γ_{th} and the rate for the optically induced heating process, given by $g_0^2 \bar{n}_- \kappa_+/\delta\Omega^2$, are smaller than the measurement rate τ_{meas}^{-1}. From the latter condition it follows that strong coupling, i.e. $g_0^2/\kappa_+\kappa_- > 1$, is also required for an undisturbed photon detection.

In the strong dispersive regime, $g_0^2 > \kappa_- \delta\Omega$, a strong projective measurement of the photon number (or analogously the phonon number) can be performed as illustrated in Fig. 3. The spectrum of the detection mode a_-, i.e. the intensity as a function of laser detuning, shows well-resolved resonances with spacing $\delta\omega$, see Fig.3(a). The weights of the peaks correspond to the photon number distribution of the signal mode. This is in close analogy to the theoretical and experimental results of [20, 21] where a qubit coupled to a microwave cavity was used to measure the photon distribution. The quantum trajectory simulations (Fig. 3(b),(c)) reveal strong measurement induced back-action leading to (anti-)correlation between signal and detection mode. Whenever the photodetector D_- registers photons from the detection mode, the state of the signal mode a_+ is projected into the zero- or one-photon Fock state depending on the detuning of the detection mode. This projection leads to a disruption of the coherent evolution of the signal mode as is clearly visible in Figs. 3(b),(c)). We note that in the regime $\tau_{\text{meas}}^{-1} > \kappa_+$, this kind of measurement backaction affects the quantum evolution significantly. Indeed, it can be shown that the photons impinging on the signal mode a_- from the coherent laser source tend to be prevented from entering the cavity due to the continuous observation of the photon number inside the cavity. This is a manifestation of the Quantum Zeno effect, as analyzed in [22].

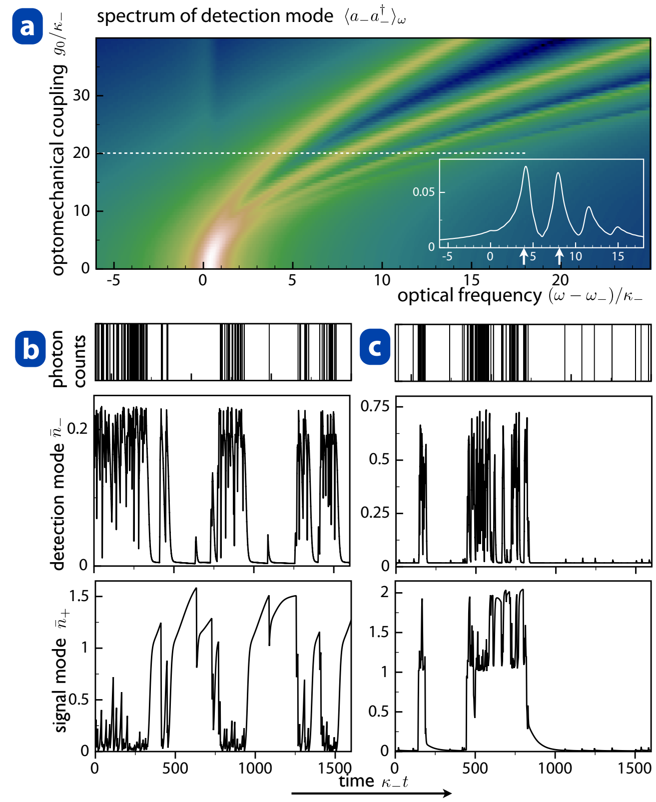

Figure 3. (a) Spectrum of the detection mode, $\langle a_- a_-^\dagger \rangle_\omega = \int e^{i\omega\tau} \langle a_-(t+\tau) a_-(t)^\dagger \rangle d\tau$, in the presence of a strongly driven signal mode, $\bar{n}_+ = 1$. With increasing optomechanical coupling rate g_0, the splitting between the resonance peaks grows like $g_0^2/\delta\Omega$. The inset shows the spectrum for $g_0 = 20\kappa_-$ (cut indicated in main figure). (b),(c) Quantum trajectories for the detection mode sitting at the (b) zero-photon resonance, $\omega_- - \omega_{L-} \approx \kappa_- \delta\omega$, and (c) at the one-photon resonance, $\omega_- - \omega_{L-} \approx 2\kappa_- \delta\omega$. This clearly shows the anti-correlation or correlation, respectively, between signal and detection modes induced by the photon interaction. $\delta\Omega = 100\kappa_-$, $\kappa_+ = 10^{-2}\kappa_-$, $\omega_{L+} = \omega_+$, $\alpha_- = \kappa_-$.

Conclusions and Outlook. - The results presented here demonstrate how the design flexibility of photonic crystals and other optomechanical systems can be exploited to significantly enhance coupling rates, and how to benefit therefrom in the deep quantum regime. Besides the dispersive QND measurement schemes based on the two-mode structure addressed here, one may think of applying the enhanced photon-photon and photon-phonon coupling for studies of optomechanical quantum many-body effects (e.g. in arrays), or for further applications in quantum information processing.

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